Levitation of the quantum Hall extended states in the $B \rightarrow 0$ limit

Th. Koschny[1,2] and L. Schweitzer[1]

[1]*Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany*
[2]*Ames Laboratory and Dept. Phys. and Astronomy, Iowa State University, Ames, Iowa 50011,*
and Foundation for Research and Technology Hellas, 71110 Heraklion, Crete, Greece
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We investigate the fate of the quantum Hall extended states within a continuum model with spatially correlated disorder potentials. The model can be projected onto a couple of the lowest Landau bands. Levitation of the $n = 0$ critical states is observed if at least the two lowest Landau bands are considered. The dependence on the magnetic length $l_B = (\hbar/(eB))^{1/2}$ and on the correlation length of the disorder potential η is combined into a single dimensionless parameter $\hat{\eta} = \eta/l_B$. This enables us to study the behavior of the critical states for vanishing magnetic field. In the two Landau band limit, we find a disorder dependent saturation of the critical states' levitation which is in contrast to earlier propositions, but in accord with some experiments.

I. INTRODUCTION

What happens to the current carrying electronic states in a quantum Hall sample when the magnetic field is turned off? According to the prevailing view, at $B = 0$ the single particle states of a disordered two-dimensional system are localized and no current can flow at zero temperatures.[1] A scenario, how this transition from the quantum Hall liquid into the insulator may take place, was suggested by Khmelnitskii[2] and Laughlin[3] 20 years ago. They both proposed that, in the limit $\omega_c = eB/m \rightarrow 0$, the extended states of the n-th Landau level float up in energy, $E_n = \hbar\omega_c (n + 1/2)(1 + (\omega_c\tau)^{-2})$, eventually crossing the Fermi level one by one, until the lowest current carrying state gets depopulated. Here, the classical elastic collision time τ is a function of the disorder strength.

Over the years, many experiments have been carried out to check this prediction, but produced only conflicting results,[4,5,6,7,8,9,10] and a convergent picture seems to become apparent only recently (see, e.g., Yasin *et al.*[11] and references therein). In essence, the levitation of the current carrying state's energies has been observed in various samples for very low magnetic fields. However, some find a saturation of the Landau level shift as $B \rightarrow 0$, for instance in high quality p-GaAs samples,[7,9] and even direct transitions to the Hall insulator from higher ($\nu > 2$) quantum Hall plateaus have been reported.[4,8,10] Therefore, the ultimate fate of E_0 in the limit $B \rightarrow 0$ remains an important and still open question. The answer to this long lasting problem has become even more pressing through the observation of the apparent 'metallic' behavior found at $B = 0$ in experiments on dilute two-dimensional electron and hole systems.[9,12,13,14]

Despite quite some effort made in the past to provide a microscopic theory,[15,16,17,18] our understanding of the levitation scenario is still limited and mainly comes from numerical studies[19,20,21,22,23,24,25] that are, however, afflicted by artifacts originating from the applied lattice model, and the difficulty of performing a proper limit $B \rightarrow 0$. The aim of our present work is to overcome these shortcomings and to study the levitation of the current carrying states within a continuum model that allows to take the essential limit $B \rightarrow 0$. Within our model, which in a first step is projected onto the lowest Lan-

dau levels (LLL), we do not find an unbounded floating up in energy, but a saturation of the quantum Hall extended states' levitation that depends on the disorder strength. The main advantages of our method are the straight forward extension to cases where additional Landau levels are taken into account as well as the possible identification of those matrix elements responsible for the levitation because they are known analytically.

II. MODEL AND METHOD

We consider non-interacting electrons moving in the x-y-plane of a two-dimensional continuum model with a perpendicular magnetic field and quenched disorder. We choose spatially correlated disorder potentials to be generated by a random weighted sum of Gaussian shaped potential hills at random sites distributed uniformly over the area of the sample. This disorder potential can be considered the continuum generalization of the correlated disorder potentials on the lattice used in our previous work[26]

$$V(\mathbf{r}) = \frac{2W}{\eta\sqrt{2\pi c}} \sum_i \exp(-|\mathbf{r} - \mathbf{r}_i|^2/\eta^2)\,\varepsilon_i. \quad (1)$$

The summation index i runs over all scatterers at random positions $\mathbf{r}_i$, the number of which is N_s. The random variables $\mathbf{r}_i$ and ε_i are both uncorrelated and uniformly distributed over the area of the system, and over the real interval $[-1,1]$, respectively. W is the strength of the disorder potential and c the concentration of the scatterers. The disorder potential is locally uniformly distributed, spatially Gaussian correlated, $<V(\mathbf{r})V(\mathbf{r}')> = (W^2/3)\exp[-|\mathbf{r} - \mathbf{r}'|^2/(2\eta^2)]$ with a correlation length $\sqrt{2}\,\eta$, has a vanishing mean $<V(\mathbf{r})> = 0$, and a uniform second moment $<V(\mathbf{r})^2> = W^2/3$. Thus, the square of the disorder strength W^2 is effectively the second moment of the disorder potential.

Now we study the Hamiltonian $H_0 = \frac{1}{2m}(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2$ of free two-dimensional electrons moving in a homogeneous perpendicular magnetic field B given by a vector potential $\mathbf{A}(\mathbf{r}) = -|B|\,y\,\mathbf{e}_x$ with $\mathbf{r} = (x, y)$ in the Landau gauge. For

a system infinite in y-direction and periodic boundary conditions in x-direction, H_0 possesses the well-known eigen base

$$\Psi_{nk}(x,y) = \tag{2}$$
$$(L_x^2 l_B^2 \pi)^{-\frac{1}{4}} \, (2^n n!)^{-\frac{1}{2}} \, H_n\left(\frac{y - \frac{\hbar k}{m\omega_c}}{2l_B^2}\right) e^{ikx}$$

in terms of the Hermite polynomials H_n. Here, L_x is the finite system width in the periodic direction, $l_B^2 = \hbar/(eB)$ the magnetic length. The non-negative integer n is the Landau level index, and $k = 2\pi m/L_x$ with integer m represents the momentum in x-direction. For the following numerical evaluation we have to restrict the momenta k to a finite number, $k \in [0, k_{\max}]$, which effectively restricts the system to a finite area $F \approx k_{\max} L_x l_B^2$ with open boundary conditions in the y-direction. We rewrite the Hamiltonian $H = \frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}(\mathbf{r})\right)^2 + V(\mathbf{r})$ for the disordered system in the representation of the free Hamiltonian's eigen base [Eq. (2)],

$$H = \sum_{n', k'; n, k} |n'k'\rangle H_{nk}^{n'k'} \langle nk| \quad \text{with} \tag{3}$$
$$H_{nk}^{n'k'} = \hbar\omega_c\left(n + \frac{1}{2}\right)\delta_n^{n'}\delta_k^{k'} + \langle n'k'|V(\mathbf{r})|nk\rangle.$$

Now the projection to a finite set of Landau levels is easily achieved by restricting the base and the n and n' sums in the Hamiltonian [Eq. (3)] to these Landau levels. Because the disorder potential is a random weighted sum over Gaussians, we can easily compute the matrix elements of the disorder potential in the approximation that the area where each single Gaussian is non-zero is small compared to the area of the sample. Then we can replace the spatial integrations over the finite sample by infinite Gaussian integrals.

Further, we introduce dimensionless quantities by measuring all energies in units of $\hbar\omega_c$ and all lengths in units of the magnetic length l_B. We will denote those quantities by attaching a hat to their symbols. This way the Hamiltonian only depends on the dimensionless correlation length parameter $\hat{\eta}$, the magnetic field only enters the energy and length scales. It takes the form

$$\hat{H}_{nk}^{n'k'} = \left(n + \frac{1}{2}\right)\delta_n^{n'}\delta_k^{k'} + \frac{2\hat{W}}{\hat{\eta}\sqrt{2\pi\hat{c}}}\sum_i^{N_s} \hat{M}_{nm}^{n'm'}(\hat{\mathbf{r}}_i)\,\hat{\varepsilon}_i.$$

To study the limit of vanishing magnetic field for a given fixed disorder potential [Eq. (1)], it is equivalent to consider the limit $\hat{\eta} \to 0$ in the dimensionless Hamiltonian. Therefore, in some respect it corresponds to a white noise disorder model which does not discern between $B = \text{const}$, $\eta \to 0$ and $\eta = \text{const}$, $B \to 0$. However, the normalization of the disorder potential may be chosen differently from the δ-property often used in studies of the high magnetic field limit.

For the disordered system projected onto the lowest two Landau levels we find the following matrix elements for a single scatterer. Inside the lowest Landau level we get with the correlation parameter $\hat{\sigma}^2 = \hat{\eta}^2 + 1 = (\eta/l_B)^2 + 1$

$$\hat{M}_{0m}^{0m'} = \frac{\sqrt{\pi}(\hat{\sigma}^2 - 1)}{\hat{L}_x\hat{\sigma}}\, e^{-\frac{\hat{\sigma}^4 - 1}{4\hat{\sigma}^2}(\frac{2\pi}{\hat{L}_x^2})^2 (m-m')^2} \times \tag{4}$$
$$e^{-\frac{(2\pi)^2}{2\hat{\sigma}^2 \hat{L}_x^2}\left((m-Y_0)^2 + (m'-Y_0)^2\right) + i\left(\frac{2\pi}{\hat{L}_x^2}\right)^2 (m-m')X_0}.$$

All higher intra and inter Landau level interaction matrix elements can be expressed as a polynomial in $m - Y_i$, $m' - Y_i$ and $m - m'$ times the intra-lowest Landau level elements. For the inter Landau level matrix elements we get

$$\hat{M}_{1m}^{0m'} = \left(\hat{M}_{0m'}^{1m}\right)^* = \tag{5}$$
$$-\frac{\sqrt{2}}{\hat{\sigma}^2}\frac{2\pi}{\hat{L}_x}\left((m - Y_i) + \frac{\hat{\sigma}^2 - 1}{2}(m - m')\right)\hat{M}_{0m}^{0m'},$$

and inside the second Landau level

$$\hat{M}_{1m}^{1m'} = \frac{2}{\hat{\sigma}^4}\hat{M}_{0m}^{0m'}\left\{\frac{\hat{\sigma}^2(\hat{\sigma}^2 - 1)}{2} + \tag{6}\right.$$
$$\left.\frac{(2\pi)^2}{\hat{L}_x^2}\left((m - Y_i)(m' - Y_i) - \frac{\hat{\sigma}^4 - 1}{4}(m - m')^2\right)\right\}.$$

Here, $Y_i = \hat{y}_i \hat{L}_x/2\pi$ and $X_i = \hat{x}_i \hat{L}_x/2\pi$ are the spatial coordinates of the particular scatterer. We write the integer m instead of the momenta $\hat{k} = 2\pi m/\hat{L}_x$ and restrict them to the range $m \in [0, N_{\hat{k}} - 1] \subset Z_1$, which renders the sample area finite. For a square sample we have $\hat{L}_y = \hat{L}_x$ and $\hat{L}_x^2 = 2\pi N_{\hat{k}}$, thus $Y_i, X_i \in [0, N_{\hat{k}} - 1] \subset R_1$. The concentration of the scatterers has been chosen in the range $c = N_s/(L_x L_y) = 10 \ldots 40$, high enough to reach the high concentration limit in the density of states.

For the Hamiltonian in the restricted n, m-base, the energy eigenvalues were now computed numerically by exact diagonalization for $N_{\hat{k}} = 48^2/2$ and various correlation length $\hat{\eta}$ and disorder strength $\hat{W}$. The positions of the extended states have been extracted by means of the level statistics method.[27,28]

III. RESULTS FOR THE TWO-LEVEL SYSTEM

A quantum Hall model projected onto the lowest Landau level only (one band model) still exhibits localization and scaling near the critical states at the band center[29]. This behavior was observed also in the presence of spatially correlated disorder potentials.[30] Of course, the one band model neither shows levitation of the critical states nor any movement of the density of states (DOS) peak position when the magnetic field or the disorder strength is varied. Therefore, we consider in the following the case of a system projected onto the lowest two Landau levels.

With increasing strength $\hat{W}$ of the disorder potential the Landau levels broaden into bands. Their width increases, with good accuracy, linearly with the disorder strength $\hat{W}$ for all correlation lengths $\hat{\eta}$ considered. The ratio of the broadening
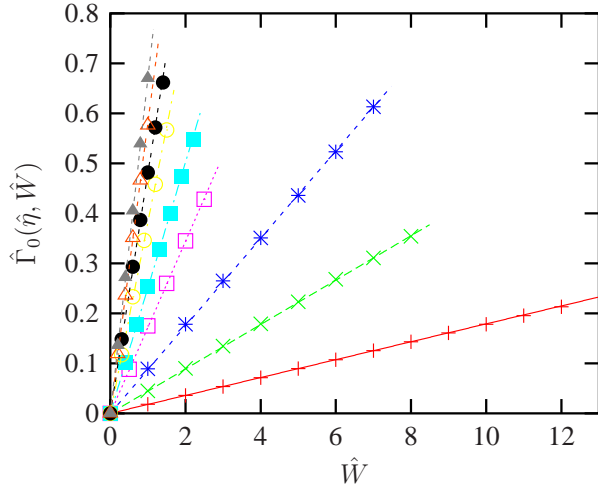
FIG. 1: (Color online) The linear broadening of the lowest Landau band's width $\hat{\Gamma}_0(\hat{\eta})$ versus the strength $\hat{W}$ of the disorder potential. The potential correlation lengths $\hat{\eta}$ are 0.02 (+), 0.05 (×), 0.1 (∗), 0.2 (□), 0.3 (■), 0.5 (○), 0.7 (●), 1.0 (△), and 1.5 (▲), respectively.

of the first and the second band increases with increasing correlation length, starting at nearly equal broadening for small $\hat{\eta}$. For a fixed second moment of the disorder potential the width of the bands increases with $\hat{\eta}$.

In Fig. 1 the full broadening $\hat{\Gamma}_0$ at half maximum is shown for the lowest Landau band. Because of its uniform behavior, we can use the width of the lowest Landau band as a measure for the effective disorder strength to compare various correla-
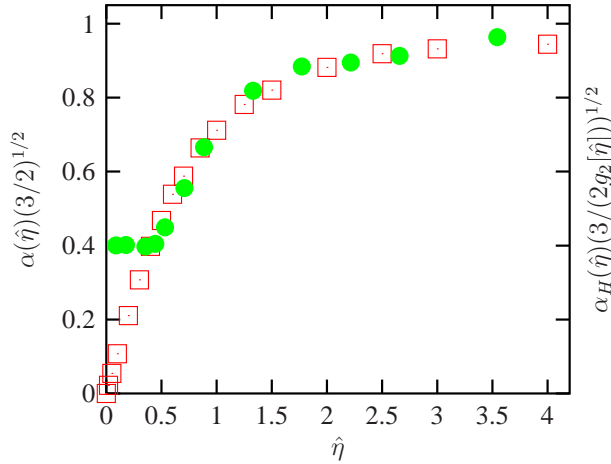

FIG. 2: (Color online) The coefficient $\alpha(\hat{\eta})$ (□) from the relation $\hat{\Gamma}_0 = \alpha(\hat{\eta})\,\hat{W}$, which collapses the linear broadening of the lowest disordered Landau band onto a single curve (see Fig. 3), is shown as a function of the reduced correlation length $\hat{\eta}$. The result from a Harper model (●) ($B = 1/8, \eta = 0.1\ldots4.0$) deviate only for $\hat{\eta} \lesssim 0.5$. To account for the lattice effects in the Harper data, actually $(3/2)^{1/2}\alpha(\hat{\eta})$ (left y-axis) and $(3/(2g_2[\hat{\eta}]))^{1/2}\alpha_H(\hat{\eta})$ (right y-axis) are shown (for $g_2[\hat{\eta}]$ see Ref. 26), which relate the width $\hat{\Gamma}_0$ of the lowest Landau band to the width of the random potential distribution for the continuum and the Harper model, respectively.
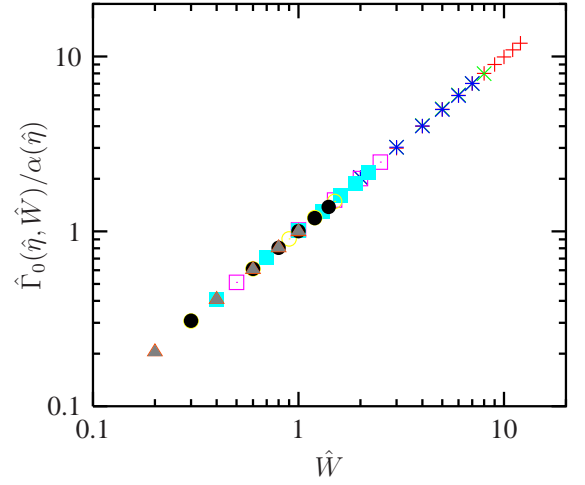

FIG. 3: (Color online) The broadening of the lowest Landau band's half width $\hat{\Gamma}_0(\hat{\eta}, W) = \alpha(\hat{\eta})\,\hat{W}$ versus disorder strength $\hat{W}$. All data for various correlation lengths $\hat{\eta}$ fall onto a straight line.

tion lengths $\hat{\eta}$.[26] Therefore we define a function $\alpha(\hat{\eta})$ which allows to collapse all $\hat{\Gamma}_0(\hat{W})$ for different $\hat{\eta}$ onto a single curve $\hat{\Gamma}_0 = \alpha(\hat{\eta})\,\hat{W}$. The function $\alpha(\hat{\eta})$ is plotted in Fig. 2 together with the numerical data obtained for the disordered Harper model for comparison. The continuum and the lattice model differ only for $\hat{\eta} \lesssim 0.5$. Note that $\alpha(\hat{\eta})$ for large $\hat{\eta}$ approaches $\sqrt{2/3}$ as has been previously reported for the Harper model.[26] Our data show a strictly linear dependence $\alpha(\hat{\eta}) \propto \hat{\eta}$ for small $\hat{\eta}$. The resulting data collapse is shown in Fig. 3.

The typical effect of increasing disorder on the the peak positions of the two Landau bands as well as on the positions of the extended states is shown in Fig. 4. The DOS peak of the lowest Landau band moves down in energy, away from the second band, which moves to higher energies with increas-
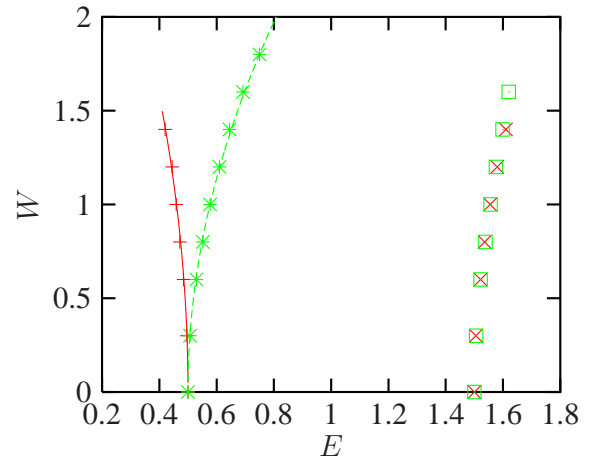

FIG. 4: (Color online) The energy E of the position of the Landau band peaks (+, ×) and of the extended states (∗, □) as a function of disorder strength W for the projected continuum model incorporating the lowest two unperturbed Landau levels. The lines are quadratic fits $\delta E \propto W^2$.
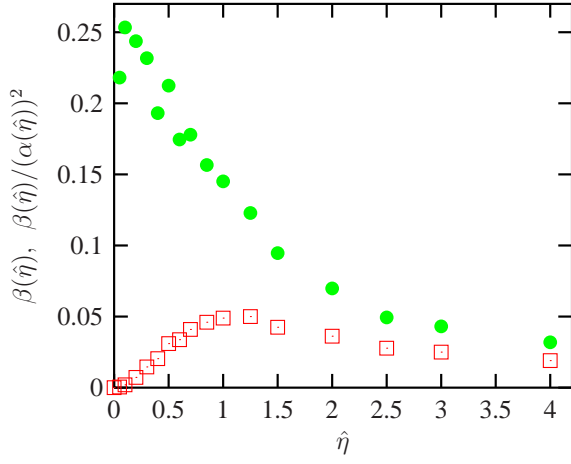
FIG. 5: (Color online) The coefficient $\beta(\hat{\eta})$ ($\square$) which describes the energy shift of the DOS peak of the lowest Landau band, $\delta\hat{E}_{\mathrm{DOS}} = -\beta(\hat{\eta})\,\hat{W}^2$, and the coefficient $\beta(\hat{\eta})/(\alpha(\hat{\eta}))^2$ ($\bullet$) relating $\delta\hat{E}_{\mathrm{DOS}}$ and $\hat{\Gamma}_0^2$. The corresponding data collapse onto a single curve is shown in Fig. 6 for various correlation lengths $\hat{\eta}$.

FIG. 7: (Color online) The levitation of the lowest Landau bands' extended states with increasing disorder strength $\delta\hat{E}_c = \gamma(\hat{\eta})\,\hat{W}^2$, collapsed onto a single curve by the function $\gamma(\hat{\eta})$, is shown for several correlation length $\hat{\eta}$.

ing disorder strength. The extended states behave differently. While those of the second band are essentially following the DOS peak, the lowest band's extended states clearly float up in energy, which can be traced approximately across 20% of the Landau gap. This behavior is qualitatively independent of the correlation length $\hat{\eta}$.

As we cannot expect the results for the second Landau band to be physically meaningful within a model projected only onto the two LLLs, we concentrate in the following on the properties of the lowest Landau band. Throughout the investigated range $\hat{\eta} = 0.05\ldots4.0$ of correlation lengths the shift of the peak position of the density of states to lower energy is with reasonable accuracy $\propto \hat{W}^2$ and of considerable magnitude, comparable with the magnitude of the extended states'

shift absolute in energy. We therefore define a function $\beta(\hat{\eta})$ which collapses all $\delta\hat{E}_{\mathrm{DOS}}(\hat{W})$ for different $\hat{\eta}$ onto a single curve $\delta\hat{E}_{\mathrm{DOS}} = -\beta(\hat{\eta})\,\hat{W}^2$. The function $\beta(\hat{\eta})$ is shown in Fig. 5, with the collapse of the data shown in Fig. 6. For a fixed second moment of the disorder potential the DOS peak shift has a maximum around $\hat{\eta} \approx 1$ and decays quadratically for $\hat{\eta} \to 0$. If the width of the lowest Landau band $\hat{\Gamma}_0$ is used as an effective disorder strength, the coefficient $\beta(\hat{\eta})/(\alpha(\hat{\eta}))^2$ of the shift of the density of states does not diverge for small $\hat{\eta}$ but rather behaves linearly, approaching a finite value for $\hat{\eta} = 0$.

The absolute levitation of the lowest Landau band's extended states with increasing disorder strength measured via $\hat{\Gamma}_0$ was calculated for several $\hat{\eta}$. The shift of the extended
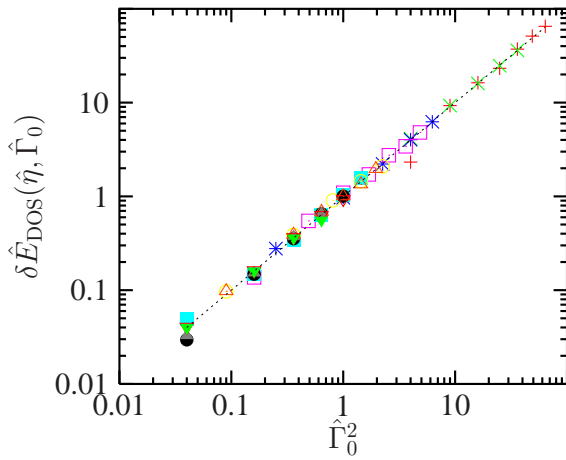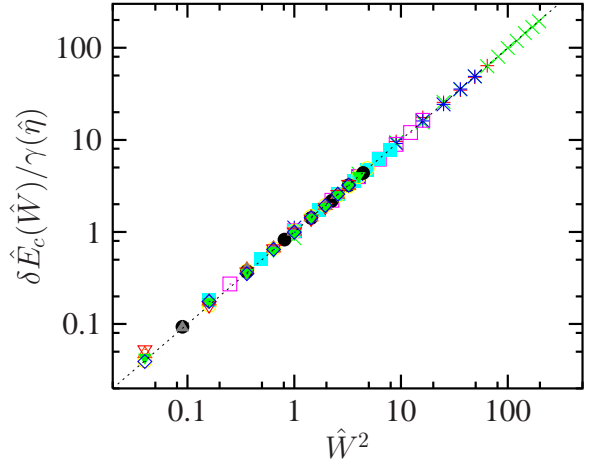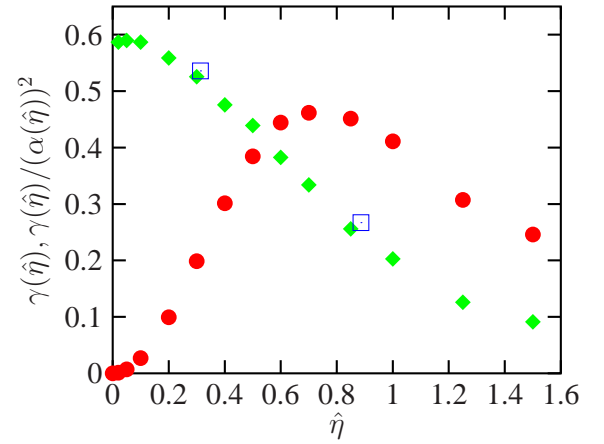

FIG. 6: (Color online) The energetic shift of the density of states' peak position versus the square of the lowest Landau level broadening $\hat{\Gamma}_0$. The data for various correlation lengths $\hat{\eta}$ fall onto a straight line, $\delta\hat{E}_{\mathrm{DOS}} = \beta(\hat{\eta})/(\alpha(\hat{\eta}))^2\hat{\Gamma}_0^2$

FIG. 8: (Color online) The coefficients $\gamma(\hat{\eta})$ ($\bullet$ multiplied by a factor 6), which collapses the data onto a straight line, and $\gamma(\hat{\eta})/(\alpha(\hat{\eta}))^2$ ($\blacklozenge$) are shown as a function of the correlation length $\hat{\eta}$. The data points ($\square$) are the corresponding results from the projected Harper model discussed below.

states' positions in energy is $\propto \hat{\Gamma}_0^2$ and thus proportional to the disorder potentials second moment $\hat{W}^2$ for all $\hat{\eta}$. Introducing a scaling function $\gamma(\hat{\eta})$, we can again collapse the data for all $\hat{\eta}$ onto a single curve $\delta\hat{E}_c = \gamma(\hat{\eta})\,\hat{W}^2$ as shown in Fig. 7. Apart from the numerically least resolved points at large disorder strength, for each $\hat{\eta}$ the data fit very well onto a single quadratic curve. We have to remark that because of the stronger broadening of the Landau bands for larger $\hat{\eta}$, larger shifts on this common curve can be resolved for smaller $\hat{\eta}$.

The functions $\gamma(\hat{\eta})$ and $\gamma(\hat{\eta})/(\alpha(\hat{\eta}))^2$ relating the levitation $\delta\hat{E}_c$ to $\hat{W}^2$ and $\hat{\Gamma}_0^2$, respectively, are shown in Fig. 8. $\gamma(\hat{\eta})$ decays quadratically for $\hat{\eta} \to 0$, shows a maximum around $\hat{\eta} \approx 0.7$, and decreases monotonically for larger $\hat{\eta}$. Therefore the levitation for a given broadening $\hat{\Gamma}_0$ of the lowest Landau band, $\gamma(\hat{\eta})/(\alpha(\hat{\eta}))^2$, is finite at $\hat{\eta} = 0$ and decreases with increasing correlation length $\hat{\eta}$.

IV. COMPARISON WITH PROJECTED HARPER MODEL

Before we start the discussion about the important limit $B \to 0$, we would like to compare our results obtained for the continuum model with those from a projected disordered Harper model.[26] The latter shows floating across almost half the Landau gap. In Fig. 9 the floating of the energetical position of the lowest extended states are shown in a similar way as in Fig. 4 for two magnetic flux densities $B = \phi_0/(64a^2), \eta = 1$, and $B = \phi_0/(32a^2), \eta = 2$, respectively. For ease of comparison, the energy scale of the Harper model has been shifted, $E = E_{\mathrm{H}} + 0.5 - E_{\mathrm{H}}^0$, where E_{H}^0 is the energy of the lowest extended states in the unperturbed Harper model, and $\phi_0 = h/e$ denotes the flux quantum and a the lattice constant.

As in the projected continuum model, the extended states float up in energy $\delta\hat{E}_c = \gamma(\hat{\eta})\,\hat{W}^2 = \gamma(\hat{\eta})/(\alpha_{\mathrm{H}})^2\,\hat{\Gamma}^2$. The perfect agreement of both models can be seen from Fig. 8 where the two data points from the Harper model ($\square$) are shown to fall onto the respective curve for the continuum model. The addition of further disorder broadened sub-bands reduce the floating of the extended states[26] and produce deviations from the $\delta\hat{E}_c \propto \hat{W}^2$ behavior which is most evident for the full disordered Harper system. We expect similar effects to occur for the continuum model when more than the two lowest Landau bands are taken into account.

V. DISCUSSION

The projection onto the lowest two Landau levels is the most simple system which shows levitation of at least the lowest Landau band's extended states. Previous work[26] has suggested that the main contribution to the levitation of the lowest level results from the coupling to the second band and should have a quadratic dependence on disorder strength. The advantages of the continuum model discussed in the present paper are (i) that we have no interfering intrinsic lattice effects as in the projected Harper models, and (ii) that we can absorb the
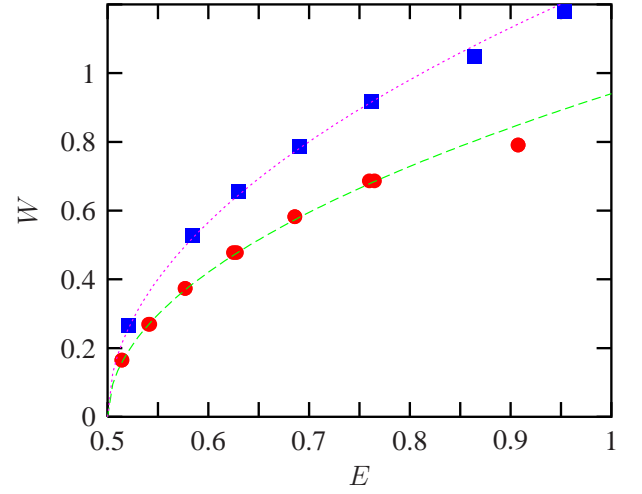

FIG. 9: (Color online) The floating of the energetical position of the lowest extended states with increasing disorder strength is shown for the two-band projected disordered Harper model. The parameters are $B = \phi_0/(64a^2)$, $\eta = 1.0a$ ($\bullet$), and $B = \phi_0/(32a^2)$, $\eta = 2.0a$ ($\blacksquare$) which correspond to $\hat{\eta} = 0.31$ and $\hat{\eta} = 0.89$, respectively. The lines are quadratic fits $\delta\hat{E} \propto \hat{\Gamma}_0^2$.

magnetic field dependence into the units of energy and length, such that the only parameters of the model are the disorder potential strength and its correlation length measured in units of the magnetic length. This reduction to a dimensionless $\hat{\eta}$-parameterized model is exact for an infinite system. By calculating the properties of the dimensionless model for decreasing $\hat{\eta}$ we can learn about the behavior of the original system for fixed disorder potential and decreasing magnetic field.

Let us now consider the limit of vanishing magnetic field. Given a fixed disorder potential, as it occurs in most experiments on samples without an additional backgate, this means the limit $\hat{\eta} \to 0$. For the width $\hat{\Gamma}_0 = \alpha(\hat{\eta})\,\hat{W}$ of the disorder broadened lowest Landau band, we find from the function $\alpha(\hat{\eta})$ at small $\hat{\eta}$ (see figure 2) a linear relation $\hat{\Gamma}_0 \sim \hat{\eta}\,\hat{W}$. In natural units this reads $\Gamma_0 \propto W\sqrt{B}$, which means that the disorder induced broadening of the Landau bands decreases for a given physical disorder potential with vanishing magnetic field B, but slower than the Landau gap. Hence, the overlap of adjacent Landau bands increases for $B \to 0$. This is qualitatively in agreement with previous work.[31,32] The levitation of the extended states $\delta\hat{E}_c = \gamma(\hat{\eta})\,\hat{W}^2$ for small B is ruled by the behavior of the function $\gamma(\hat{\eta})$ near $\hat{\eta} \approx 0$. Since $\gamma(\hat{\eta}) \sim \hat{\eta}^2$ for small $\hat{\eta}$, the coefficient $\gamma(\hat{\eta})/(\alpha(\hat{\eta}))^2 = \mathrm{const} + o(\hat{\eta})$ for the levitation of the extended states $\delta\hat{E}_c \propto \hat{\Gamma}_0^2$ starts as a constant (see Fig. 8). Hence, the shift of the extended states' energy is confined by the broadening of the Landau levels DOS, $\delta\hat{E}_c \approx \mathrm{const} \cdot \hat{\Gamma}_0^2$.

In experiments, where the second moment of the disorder potential is fixed in natural units, the relative broadening $\hat{\Gamma}_0$ diverges for small magnetic field $\hat{\Gamma}_0 \sim W/\sqrt{B}$ which leads to an infinite shift of the relative Energy $\delta\hat{E}_c \sim W^2/B$ of the lowest Landau band's extended states. As a consequence the critical filling factor, i.e., the number of electrons divided by

the number of flux quanta penetrating the area of the system where the Fermi energy coincides with the energy of the extended states, diverges as well for $B \to 0$. However, if we consider the energy shift δE_c of these extended states in natural units, the B-dependence cancels for small B and we find with $\hat{\eta}^2 \propto \eta^2 B$ a *finite* levitation $\delta E_c = \gamma(\hat{\eta})/(\hbar\omega_c) W^2 \sim \text{const} \cdot W^2$ that only depends on the properties of the disorder potential. This result clearly conflicts with the levitation scenario proposed by Khmelnitskii[2] and Laughlin[3] which predicts a divergent energy of the extended states for vanishing magnetic field. In order to obtain such a divergency from our model, the function $\gamma(\hat{\eta})/(\alpha(\hat{\eta}))^2$ would have to diverge like $o(\hat{\eta}^{-2})$, hence $\gamma(\hat{\eta}) \sim \text{const}$, for small $\hat{\eta}$. Here, we assumed the scattering time τ in the Khmelnitskii-Laughlin expression for the energy level float up to be independent of B. There are, however, attempts to describe the classical low field magnetoresistance in terms of a B-dependent τ.[33,34,35]

Interpreting these results we have to consider several issues. First of all, in the present work we included only the lowest two Landau bands. Though this is certainly reasonable for small broadening of the Landau bands because the strongest influence on the first Landau band's extended states comes from the inter-band matrix elements that couple the first to the second Landau level. Yet, it is not clear that the influence of the higher levels remains small for stronger overlapping bands. From projected disordered Harper models including the lowest two and the lowest three Harper bands,[26] we have some evidence that the addition of the third band reduces the levitation of the first band's extended states. The dependence on the disorder strength remains quadratic in this case. On the other hand for strong magnetic fields, the levitation of the lowest band's extended states in full disordered Harper models has shown to be even slower and not quadratic in the disorder strength anymore. However, for those models the levitation of the extended states could be traced at least across the Landau gap, which is far beyond the resolvable range in the continuum model discussed in the present paper. Nevertheless, it remains to be seen, whether the finite levitation result obtained here will persist when taking into account more than the lowest two Landau bands.

Second, the possible influence of the finite system size. The elimination of the explicit B dependence of the system by the reduction to the $\hat{\eta}$-parameterized dimensionless model is only exact for an infinite system. For the numerical simulation we have to limit the Hilbert space which is done by restricting the number of Landau levels n and, particularly, the number of momenta k in the representation of the projected Hamiltonian [Eq. (3)]. The limitation of the k effectively renders the area of the sample, $F \approx k_{max} L_x l_B^2$, finite and the dimensionless model an approximation. However, the finite size of the sample is expected to affect only large $\hat{\eta}$, or large η for a given B, not the opposite limit $\hat{\eta} \to 0$ which we are discussing here. Note that the effective (linear) size of the system scales with l_B. The other approximation we made with respect to the system size is the restriction of the spatial integration domain over the Gaussian scattering potentials in the calculation of the matrix elements. We approximated the finite integration domain in the numeric simulation by indefinite Gaussian inte-

grals. Again this should not affect the limit of short correlation lengths. In a finite sample the simulation results will depend on the concentration c of the scatterers in the random potential [Eq. (1)] albeit the local statistical properties of the potential do not. In our calculations we have chosen increasing c for smaller $\hat{\eta}$, high enough to reach the high-concentration limit in the density of states. Because of the limited available computer power it was not feasible to directly study the anticipated corresponding convergence in the behavior of the critical states with increasing scatterer concentration. This should be addressed in future work.

Third, the behavior has been extrapolated from relatively small but non-zero $\hat{\eta}$. Although there is no indication for a discontinuous behavior when $\hat{\eta} \to 0$, there is no guarantee that we can extrapolate our data that far. However, the range of computed $\hat{\eta}$ seems to be comparable with experiments. For instance, combining atomic force microscopy with selective etching, the image of the topology of interfaces in AlGaAs/GaAs quantum well structures was shown to exhibit smoothly varying structures with correlation length in the range $4 \cdot 10^{-8}$ m to $2 \cdot 10^{-7}$ m.[36] For the latter value, this leads to approximately $0.1 \leq \hat{\eta} \leq 10$ when the magnetic flux density is varied between 10^{-4} and 1 Tesla.

Our findings for the disorder broadening of the lowest Landau level are in qualitative agreement with results obtained previously by Ando and Uemura.[31] The basic difference to our work is the incompatible definition of the disorder potential: Our $V(r)$ is normalized to a given second moment, whereas Ando and Uemura normalize their disorder potential to a fixed integral $\int V(r) \, d^2r$ (δ-property). This leads to an additional factor $\hat{\eta}$ in the potential strength, $\hat{W}_{AU} \propto \hat{\eta} \hat{W}$. In the limit of small correlation length $\hat{\eta} \to 0$, we find for the broadening of the lowest Landau level a linear $\hat{\eta}$-dependence, $\Gamma_0 \sim \hat{\eta} W$, which directly translates into Ando and Uemura's independence on the correlation length, $\Gamma_0 \sim W_{AU}$. In the opposite limit of large $\hat{\eta}$, we find that the broadening of the DOS just corresponds to the second moment of the disorder potential, again perfectly in agreement with their results. For the behavior of the critical states in the limit $B \to 0$, not considered by Ando and Uemura, the alternative normalization of the potential strength may lead to significantly different results. In our case, we consider it physically correct to extract the B-dependence of the energy of the critical states from the $\hat{\eta}$-dependence of the model with a fixed second moment $W^2/3$ of the disorder potential in natural units. If we used instead the $\hat{\eta}$-dependence for a preserved δ-property of the potential, i.e., $\int V(r) \, d^2r = \text{const}$ as for the potential used by Ando and Uemura,[31] we would obtain a singular dependence $\delta E_c \propto W_{AU}^2/B$ for $B \to 0$ which corresponds better to the levitation scenario proposed by Khmelnitskii and Laughlin.

VI. SUMMARY

We investigated a disordered two-dimensional continuum model for the integer quantum Hall effect projected onto the lowest two Landau levels. The chosen dimensionless repre-

sentation enabled us to make statements about the energetic shift of both, the magnetic field and correlations length dependence of the density of states peak and of the extended states. The results were obtained from a numerical investigation of the model, parameterized by the strength $\hat{W}$ and the correlation length $\hat{\eta}$ of an effective disorder potential only. The magnetic field B dependence was entirely absorbed into the units of length and energy.

For the two-level continuum model we found qualitatively the same behavior as has been observed for disordered Harper models projected onto the lowest two sub-bands.[26] With increasing disorder strength $\hat{W}$ the Landau bands broaden linearly. We clearly observed floating up in energy of the lowest Landau band's extended states while the peak of the density of states moves downwards. Both effects are proportional to the square of the disorder strength.

In the limit $B \rightarrow 0$, which is difficult to access in numerical studies on lattice models, we find for the projected two-band continuum model that the energy of the extended states only floats up to a finite value that depends on disorder. As this result is at variance with the diverging floating scenario suggested 20 years ago[2,3], we discuss a few limitations of our work that may be responsible for the discrepancy. On the other hand, our result is in accord with some experimental work. Still, the addition of higher Landau levels and the investigation of the relevant matrix elements are clearly needed to settle this issue and to gain a deeper insight into the microscopic origin of the levitation of the extended states and the corresponding quantum Hall to insulator transition. We feel that to follow up our approach within a projected continuum model will render this objective possible.

VII. ACKNOWLEDGMENTS

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